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T.B.C. : BAC–90

Test Booklet Series

Serial No. 603112

D

TEST BOOKLET

MATHEMATICS

Time Allowed : 2 Hours

Maximum Marks : 300

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7. All items carry equal marks. All items are compulsory. Your total marks will depend only on the number of correct responses marked by you in the Answer Sheet. Four each question for which a wrong answer is given by you, **one fifth (0·20) of the marks assigned to that question will be deducted as penalty.**
8. Before you proceed to mark in the Answer Sheet the responses to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your **Admission Certificate**.
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1. Let $\alpha = \overline{\lim_{n\to\infty}} x_n$ and $\beta = \underline{\lim_{n\to\infty}} x_n$ where $x_n = \frac{n-1}{n+2} \cos \frac{2\pi n}{3}$, then (α, β) is

(a) $(1, -1)$

(b) $\left(1, \frac{1}{2}\right)$

(c) $(1, 0)$

(d) $\left(1, -\frac{1}{2}\right)$

2. Let $O_n = \left(\frac{1}{n+1}, \frac{1}{n}\right)$ be an open set of real numbers, then sequence $\{O_n\}_{n=1}^{\infty}$ forms a

(a) countable cover of (0, 1)

(b) countable cover of [0, 1)

(c) countable cover of [0, 1]

(d) countable cover of (0, 1]

3. The series $\sum_{n=1}^{\infty} 3^{n-(-1)^n}$ is

(a) Oscillatory convergent

(b) Convergent (having all positive terms)

(c) Oscillatory Divergent

(d) Divergent to infinity

(Oscillatory means terms are alternatively + & –)

4. Which one of the following set is uncountable ?

(a) set of prime numbers

(b) set of all rational numbers

(c) set of all algebraic numbers

(d) set of all real numbers in (0, 1)

5. Let $\phi(x) = |x|$ for $0 < |x| \leq 2$ and equal to 1 at x = 0, then the point x = 0 corresponds to

(a) local maximum point of ϕ

(b) local minimum point of ϕ

(c) neither maximum nor minimum point of ϕ

(d) global maximum point of ϕ over [–2, 2]

6. The straight line $lx + my = 1$ is normal to the curve $y^2 = 4ax$ if

(a) $al^3 + 2alm^2 = m$

(b) $al^3 + 2alm^2 = m^2$

(c) $al^3 - 2alm^2 = m^2$

(d) $al^3 + 2alm^2 = -m^2$

7. The number of rows necessary in the construction of truth table of $p \wedge (q \vee r)$ is

(a) 4

(b) 8

(c) 9

(d) none of these

8. The proposition $p \vee \sim (p \wedge q)$ is

(a) always a tautology

(b) always a contradiction

(c) a tautology only for some values of p and q

(d) none of these

9. If $\downarrow$ denotes the connective, which of the following is correct ?

(a) $p \vee q \equiv (p \downarrow p) \downarrow (q \downarrow q)$

(b) $p \wedge q \equiv (p \downarrow p) \downarrow (q \downarrow q)$

(c) $p \sim q \equiv (p \downarrow p) \downarrow (q \downarrow q)$

(d) none of these

10. For a universal set A = {1, 2, 3, 4}, the truth value of the following three statements

(i) $\forall x, x + 3 < 6$

(ii) $\exists x, x + 3 < 6$

(iii) $\exists x, 2x^2 + x = 15$

is

(a) True for (i), (ii) and (iii)

(b) False for (i), (ii) and (iii)

(c) True for (i) and (ii), but False for (iii)

(d) False for (i) and (iii), but True for (ii)

11. Which of the following is correct ?

(a) $p \vee q$ logically implies $p \leftrightarrow q$

(b) $p \sim q$ logically implies $p \leftrightarrow q$

(c) $p \wedge q$ logically implies $p \leftrightarrow q$

(d) none of the foregoing

12. The argument : $p \rightarrow q, q \vdash p$ is

(a) valid for all values of p and q

(b) a fallacy for all values of p and q

(c) valid only for specific values of p and q

(d) none of these

13. If S be a set of 9 elements, then the number of elements in the power set of S is

(a) 9

(b) 81

(c) 512

(d) 729

14. A, B and C are three sub-sets of a universal set S. Then which one of the following is wrong ?

(a) $(A \cup B)' = A' \cap B'$

(b) $[A \cap (B \cup C)] \cap [A' \cap (B' \cap C')] = \phi$

(c) $A \Delta C = C \Rightarrow A \cap (B \cup C) = C$

(d) none of these

15. Among the following which is not correct ?

(a) $Q^+ \subseteq R$

(b) $R^+ \subseteq Q$

(c) $Z^+ \subseteq Q$

(d) $R^+ \cap C = R^+$

16. A sample of 80 vehicle owners revealed that 24 owned station wagons, while 62 owned cars. Then the number of people who owned both a station wagon and a car, is

(a) 38

(b) 18

(c) 6

(d) 56

17. Identify which of the following sets is infinite

(a) {sides of a cube}

(b) {lines that satisfies 3x = y}

(c) {squares with the points (0, 0), (0, 1), (0, 4) as corners}

(d) {lines through the origin}

18. Suppose12 people read the Statesman or the Times of India or both. Given 3 people read only the Statesman and six read both, find the number of people who read only the Times of India

(a) 21

(b) 9

(c) 3

(d) 15

19. The value of the determinant $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix}$ is

(a) (a – b) (b – c) (c – a)

(b) (a – b) (b + c) (c – a)

(c) 0

(d) none

20. If $A=\begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{pmatrix}$, then the sum of all elements of A^{-1} is

(a) $\frac{2}{7}$

(b) $-\frac{3}{7}$

(c) $\frac{3}{7}$

(d) 0

21. The rank of the matrix $A=\begin{pmatrix} 1 & 0 & 3 \\ 4 & -1 & 5 \\ 2 & 0 & 6 \end{pmatrix}$ is

(a) 1

(b) 2

(c) 3

(d) none of these

22. A non-singular matrix P commutes with P^T. Then

(a) P is orthogonal

(b) $P^{-1}P^T$ is orthogonal

(c) $P + P^T$ is orthogonal

(d) none of the foregoing

23. If $A=\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, then

(a) $A^2 + 4A - 5I_3 = 0$

(b) $A^2 - 4A + 5I_3 = 0$

(c) $A^2 - 4A - 5I_3 = 0$

(d) none of these

24. If H = P + iQ be a Hermitian matrix, then

(a) the diagonal elements of H are imaginary

(b) P is a real symmetric matrix and Q is a real skew-symmetric matrix

(c) P is a real skew-symmetric matrix and P is a real symmetric matrix

(d) none of these

25. If a and a^2 are both generators of a cyclic group of order n, then

(a) n must be odd

(b) n must be even

(c) n must be prime

(d) n must not be prime

26. In a group (G, ∗), a is an element of order 30. Then the order of a^{18} is

(a) 540

(b) 18

(c) 5

(d) none of these

27. The domain of the function

$$f(x) = \sin^{-1}\frac{1}{2}(x-3) - \log_{10}(4-x)$$

is

(a) $(-\infty, 3)$

(b) $[1, 4)$

(c) $[1, 4]$

(d) $(-\infty, 3]$

28. Let the function $f(x) = 1 + |x|$ for $x < -1$ and is equal to $[x]$ when $x \geq -1$, where $[x]$ is the greatest integer $\leq x$. Then $f(f(-2 \cdot 3))$ is equal to

(a) 3

(b) –3

(c) 2

(d) –2

29. The relation $x' = \frac{x}{1+x}$ transforms the unbounded interval $[0, \infty)$ into

(a) unbounded interval $(-\infty, 0)$

(b) unbounded interval $(1, \infty)$

(c) bounded interval $[0, 1)$

(d) bounded interval $[0, 1]$

30. The functions $f(x) = \frac{1}{2}\sin^2 x + \frac{1}{2}\cos^2 x$ and $g(x) = \sec^2 x - \tan^2 x$ are equal over the set

(a) $\mathbb{R}$, set of real numbers

(b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R} - \{x : x = 2n\pi\}$

(d) $\mathbb{R} - \{x : x = n\pi + \frac{\pi}{2}, n \in I\}$

where I denotes the set of integers.

31. Range of the function

$$f(x) = \cos^{-1}(2 - x) \text{ is}$$

(a) $[0, \pi]$

(b) $\left[-\frac{\pi}{2}, 0\right] \cup \left[\frac{\pi}{2}, \pi\right]$

(c) $\left[0, \frac{\pi}{2}\right] \cup [0, 1]$

(d) $\left[\frac{\pi}{2}, \pi\right] \cup \left[-\frac{\pi}{4}, 0\right]$

32. Let

$S = \{(x, y) : x, y \in \mathbb{R} \text{ and } x^2 + y^2 \leq 25\}$ and

$T = \{(x, y) : x, y \in \mathbb{R} \text{ and } y \geq \frac{4}{9}x^2\}$ be two relations on the set of real numbers. Let the domain D of $S \cap T$ is the set $\{x : x \text{ is real}, x \in [\alpha, \beta]\}$ where $[\alpha, \beta]$ is

(a) $(-5, 5)$

(b) $[-3, 3]$

(c) $[-5, -3]$

(d) $(3, 5]$

33. Let $\alpha = \lim_{x \to 5+} \frac{x^2 - 9x + 20}{x - [x]}$ and

$\beta = \lim_{x \to 5-} \frac{x^2 - 9x + 20}{x - [x]}$, then

(a) α exists but β does not exist

(b) $\alpha = 1$, $\beta = 0$ (both exist)

(c) α does not exist, $\beta = 0$

(d) α and β (both) do not exist

34. Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^{\circ}$ such that $f(x, y) = \sqrt{x^2 + y^2}$, then f is

(a) many-one onto

(b) one-one onto

(c) one-one into

(d) many-one into

where $\mathbb{R}^{\circ}$ denotes the set of nonnegative real numbers.

35. The integer just greater than $(1 + 0{\cdot}0001)^{10000}$ is

(a) 2

(b) 4

(c) 5

(d) 3

36. If x and y are rational numbers such that $x + y + \sqrt{2}(x - 2y) = 2x - y + \sqrt{6}(x - y - 1)$, then

(a) $x = 2, y = 1$

(b) $x = 5, y = 1$

(c) $x = \frac{1}{2}, y = 1$

(d) x & y cannot be determined

37. The modulus r and the principal argument θ of the complex number $z = (\tan 1 - i)^2$ is

(a) $r = \sec^2 1;\ \theta = 2 - \pi$

(b) $r = \sec^2 1;\ \theta = -\frac{\pi}{4}$

(c) $r = \sec^2 1;\ \theta = \frac{\pi}{2}$

(d) $r = \sec^2 1;\ \theta = \frac{\pi}{4}$

38. The locus of the point z given by the relation

$$\text{Real part}\left(\frac{1}{z}\right) < \frac{1}{2} \text{ is}$$

(a) Interior of the circle with centre (1, 0) and radius 1

(b) Interior of the ellipse with centre (0, 0), major axis 1 and minor $\frac{1}{2}$

(c) the exterior of the circle with centre (1, 0) and radius 1

(d) exterior of the ellipse with centre (0, 0), major axis 1 and minor axis 1/2

39. Let G be a finite abelian group of odd order and let

$H = \{x^2 / x \in G\}$.

Then

(a) H is a subgroup of G only if G is cyclic

(b) H is a proper subgroup of G

(c) H = G

(d) H may not be a subgroup of G

40. Which one of the following is wrong ?

(a) (Z, +) is a semigroup

(b) (Q, ·) is group

(c) (Q, +) is a commutative group

(d) (R, +) is a group

41. If G be a group of order 8, then

(a) G is commutative

(b) all proper subgroups of G are commutative

(c) G is commutative and all proper subgroups of G are commutative

(d) none of these

42. Which one of the following is wrong ?

(a) group of order less than 6 is commutative

(b) groups of prime orders are cyclic

(c) a commutative group is cyclic

(d) the order of each element in a finite group G is a divisor of O(G)

43. $R = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix}, \text{a, b real numbers} \right\}$.

Then under matrix addition and matrix multiplication, R is

(a) not a ring

(b) a non-commutative ring

(c) a commutative ring but not a field

(d) a field

44. Which one of the following is wrong ?

(a) The characteristic of an integral domain R is either zero or prime

(b) A field is an integral domain

(c) An integral domain is a field

(d) None of these

45. In the ring Z of integers, find which of the following subset of Z are sub-rings

(a) The set of integers of the form $4K + 2, K \in Z$

(b) The set of integers of the form $4K + 1, K \in Z$

(c) The set of integers of the form $4K, K \in Z$

(d) None of these

46. Which one of the statements is correct ?

(a) Rings R and Q are isomorphic

(b) Rings R and C are isomorphic

(c) Rings Z_6 and $Z_3 \times Z_2$ are isomorphic

(d) None of these

47. Let V(R) be the vector space of all real valued functions of R. Which one of the following is not a subspace of V ?

(a) The set of all constant functions in V

(b) The set of all functions f in V such that $f(0) = 0$

(c) The set of all polynomials of degree 3

(d) The set of all polynomial functions on V

48. The four vectors (1, 1, 0, 0), (1, 0, 0, 1), (1, 0, a, 0) and (0, 1, a, b) are linearly independent if

(a) $a \neq 0, b \neq 2$

(b) $a \neq 2, b \neq 0$

(c) $a \neq 0, b \neq -2$

(d) $a \neq -2, b \neq 0$

49. In $R^3, \vec{\alpha}=(4,3,5)$ $\vec{\beta}=(0,1,3)$, $\vec{\gamma}=(2,1,1)$ and $\vec{\delta}=(4,2,2)$, then

(a) $\vec{\beta}$ is a linear combination of $\vec{\gamma}$ and $\vec{\delta}$

(b) $\vec{\alpha}$, $\vec{\beta}$ and $\vec{\gamma}$ are linearly independent

(c) $\vec{\alpha}$ is a linear combination of $\vec{\beta}$ and $\vec{\gamma}$

(d) none of these

50. Let $\{\alpha, \beta, \gamma\}$ be a basis of a real vector space V and C be a non-zero real number. Which one of the following is wrong ?

(a) $\{c\alpha, c\beta, c\gamma\}$ is a basis of V

(b) $\{\alpha + c\beta, \beta, \gamma\}$ is a basis of V

(c) $\{\alpha + c\beta, \beta + c\gamma, \gamma + c\alpha\}$ is a basis of V

(d) none of these

51. Let A, B be the points (a, 0), (b, 0) respectively and let AC, BC be the straight lines $y = M(x - a)$ and $y = -M(x - b)$ respectively. Suppose f(x) is a differentiable function over [a, b] such that $f(a) = 0 = f(b)$ and $|f'(x)| \le M$ for all x, $a \le x \le b$ where $'$ denotes the derivative of f(x) w.r. to x.

If $\left|\int_a^b f(x)\,dx\right| < \frac{M}{4}(b-a)^{\alpha}$, then α is

(a) 2

(b) 3

(c) 1

(d) 4

52. Area of the region bounded by the curve $y = 3 - |x|$ and x-axis is

(a) 5

(b) 6

(c) 9

(d) 3

53. The numerically greatest term in the expansion of $(1 - x)^{-14/3}$ when $x = \frac{21}{32}$ is

(a) 5th and 6th terms

(b) 6th and 7th terms

(c) 7th and 8th terms

(d) 8th and 9th terms

54. What is the first negative coefficient in the expansion of $\left(1+\frac{2x}{3}\right)^{21/4}$ where $x > 0$?

(a) $-\frac{3\cdot1\cdot5\cdot9\ldots\ldots\ldots21}{7!\,6^7}$

(b) $-\frac{5\cdot3\cdot1\cdot3\cdot7\ldots\ldots\ldots21}{5!\,6^5}$

(c) $-\frac{1\cdot5\cdot7\cdot11\ldots\ldots\ldots21}{9!\,6^9}$

(d) $-\frac{3\cdot5\cdot7\ldots\ldots\ldots21}{7!\,6^7}$

55. In how many ways one can make a first, second, third and fourth choice among 12 firms leasing construction equipment ?

(a) 11,870

(b) 11,880

(c) 11,890

(d) 12,000

56. If A and B are mutually exclusive events $P(A) = 0{\cdot}29$, and $P(B) = 0{\cdot}43$, then $P(\overline{A} \cap \overline{B})$ is

(a) 0·26

(b) 0·27

(c) 0·28

(d) 0·29

57. The value of θ in the first mean value theorem

$f(x + h) = f(x) + hf'(x + \theta h)$, if $f(x) = ax^2 + bx + c$, $a \neq 0$ is

(a) $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) 1

(d) 0

58. Rolle's theorem can be applicable to the function

(a) $|\sin x|$, $x \in (0, 2\pi)$

(b) $\sin \frac{1}{x}$, $x \in (-1, 1)$

(c) $x^2 - 3x + 2$, $x \in (1, 2)$

(d) $|x|$, $x \in (-1, 1)$

59. Let $y = 2x + 3$ is a tangent to the parabola $y^2 = 24x$, then its distance from the parallel normal is

(a) $5\sqrt{5}$

(b) $15\sqrt{5}$

(c) $10\sqrt{5}$

(d) $\sqrt{5}$

60. In order that the function $f(x) = (x + 1)^{\cot x}$ be continuous at $x = 0$, $f(0)$ must be defined as

(a) $\frac{1}{e}$

(b) ∞

(c) 1

(d) e

61. The function $f(x) = \frac{x}{\log_e x}$ decreases in

(a) (e, ∞)

(b) $(0, 1) \cup (1, e)$

(c) $(0, 1]$

(d) $(1, \infty)$

62. If $z = x \sin^{-1}\frac{x}{y} + y\cos^{-1}\frac{y}{x}$ satisfies the equation

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = B,$$

then B is

(a) 0

(b) 1

(c) $\sin^{-1}\frac{x}{y} + \cos^{-1}\frac{y}{x}$

(d) 2

63. Let

$$I_1 = \int_{1-K}^{K} x f(x(1-x))\,dx \text{ and}$$

$$I_2 = \int_{1-K}^{K} f(x(1-x))\,dx$$

where $2K - 1 > 0$, then $\frac{I_1}{I_2}$ is

(a) $\frac{1}{3}$

(b) $\frac{1}{4}$

(c) 1

(d) $\frac{1}{2}$

64. Let $I = \int_0^1 e^x\left[\tan^{-1}x + \frac{1}{1+x^2}\right]dx$, then I is equal to

(a) $\frac{\pi}{4}$

(b) πe

(c) $\frac{\pi e}{4}$

(d) $\frac{\pi e}{2}$

65. If $I = \int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}}dx$, then I is equal to

(a) 0

(b) $\frac{3}{2}$

(c) 1

(d) $\frac{1}{2}$

66. If $\int_0^1 e^x (x-1)^n\,dx = 16 - 6e$, where n is a positive integer ≤ 5, then the value of n is

(a) 4

(b) 3

(c) 2

(d) 5

67. What is the radius of the circle given by

$x^2 + y^2 + z^2 = 25$,

$x + 2y + 2z + 9 = 0$?

(a) 1 unit

(b) 2 units

(c) 3 units

(d) None of the above

68. The vertices of a triangle has position vectors $2\hat{i} + 4\hat{j} - \hat{k}$, $4\hat{i} + 5\hat{j} + \hat{k}$ and $3\hat{i} + 6\hat{j} - 3\hat{k}$. Then the triangle is

(a) right angled

(b) isosceles

(c) equilateral

(d) None of the above

69. For any three vectors $\vec{a}, \vec{b}, \vec{c}$, the scalar $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$ is equal to

(a) $2[\vec{a}\ \vec{b}\ \vec{c}]$

(b) $[\vec{a}\ \vec{b}\ \vec{c}]$

(c) $[\vec{a}\ \vec{b}\ \vec{c}]^2$

(d) None of the above

70. What is the angle between the vectors

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$?

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$

(d) None of the above

71. In a balance, we measure

(a) mass

(b) weight

(c) momentum

(d) gravitational mass

72. Law of parallelogram of forces holds for

(a) coplanar forces

(b) non-coplanar forces

(c) both coplanar and non-coplanar forces

(d) all kinds of forces

73. Angular momentum acts

(a) in the plane of the acting force

(b) in the plane of force but in the direction opposite to the force

(c) perpendicular to the axis of rotation

(d) along the axis of rotation

74. Find the correct statement in the following

(a) Torque is the rate of change of angular momentum

(b) Torque is moment of the force

(c) Torque causes rotational motion

(d) Statements (a), (b) and (c) are true

75. Motion

(a) is a continuous change of position of a particle

(b) is a continuous change of position of a particle with respect to a reference point

(c) is change of position of a particle

(d) can not be defined

76. D'Alembert's principle

(a) is the principle of virtual work

(b) is generalization of the principle of virtual work

(c) is generalization of the principle of virtual work for dynamical systems

(d) acts at a particular instant

77. During harmonic motion

(a) motion of a particle is simple

(b) particle goes through equal distances on both sides of the equilibrium position

(c) particle does not go through equal distances on both sides of the equilibrium position

(d) None of the above statement is true

78. Projectile gets maximum horizontal range when it is thrown at an angle

(a) 45°

(b) 30°

(c) 90°

(d) between 30° and 45°

79. The equation $x^4 - 4x^3 + ax^2 + 4x + b = 0$ has two pairs of equal roots. Then the values of a and b are

(a) a = 2, b = 1

(b) a = 1, b = 2

(c) a = –1, b = 2

(d) a = 1, b = – 2

80. A root of the equation $x^3 - 9x + 1 = 0$ correct to three decimal places using Bisection method is

(a) 2·349

(b) 2·439

(c) 2·493

(d) 2·943

81. The bacteria concentration in reservoir varies as $C = 4e^{-2t} + e^{-0\cdot 1t}$. The time required for the bacteria concentration to be 0·5 is (by using Newton-Raphson Method)

(a) 6·889

(b) 6·988

(c) 6·898

(d) 6·888

82. The observed values of a function are respectively 168, 120, 72 and 63 at the four positions 3, 7, 9 and 10 of the independent variable. Then the best estimate for the value of the function at the position 6 is

(a) 141

(b) 143

(c) 145

(d) 147

83. A curve is given by the table

x	y
0	0
1	2
2	2·5
3	2·3
4	2
5	1·7
6	1·5

The x-coordinate of the centre of gravity of the area bounded by the curve, the end ordinates and the x-axis is given by

$$A\bar{x} = \int_0^6 xy\,dx, \text{ where A is the area.}$$

Then $\bar{x}$ is (using Simpson's rule)

(a) 3·302

(b) 3·203

(c) 3·032

(d) 3·230

84. The solution of $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$ for $x = 0{\cdot}2$ by Runge-Kutta second order method is

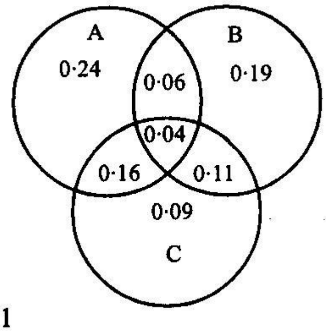
(a) 0·204

(b) 0·240

(c) 0·024

(d) 0·042

85. With reference to the accompanying figure, the value of $P(A \cap B \cap C \mid B \cup C)$ is



(a) $\frac{1}{65}$

(b) $\frac{2}{65}$

(c) $\frac{3}{65}$

(d) $\frac{4}{65}$

86. In a bag there are six balls of unknown colours, three are drawn and found to be black. The probability of no black ball left in the bag is

(a) $\frac{1}{35}$

(b) $\frac{2}{35}$

(c) $\frac{3}{35}$

(d) $\frac{4}{35}$

87. $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = c^2 \left(\frac{d^2y}{dx^2}\right)^2$ is of

(a) second degree and second order

(b) second degree and third order

(c) first degree and sixth order

(d) none of these

88. For the differential equation

$$Mdx + Ndy = 0$$

to be exact

(a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(c) $\frac{\partial M}{\partial x}$ and $\frac{\partial N}{\partial y}$ exist

(d) $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$ exist

89. For the differential equation

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$$

integrating factor is

(a) e^{-x}

(b) $(x + 1)$

(c) $(x + 1)^2$

(d) $\frac{1}{x+1}$

90. If the differential equation

$$Mdx + Ndy = 0$$

be homogeneous in x and y then

(a) $\frac{1}{Mx - Ny}$ is integrating factor

(b) $Mx - Ny$ is integrating factor

(c) $\frac{1}{Mx + Ny}$ is an integrating factor

(d) $Mx + Ny$ is an integrating factor

91. Solution of

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$$

is y equals

(a) $Ae^x + Be^{-x} + Ce^{2x}$

(b) $(A + Bx)e^x$

(c) $(A + Bx)e^x + Ce^{2x}$

(d) none of the above

92. Particular integral of

$$\left(\frac{d^2}{dx^2} + 4\right)y = \sin^2 x \text{ is}$$

(a) $\frac{x}{8}\sin 2x$

(b) $\frac{x}{8}\cos 2x$

(c) $\frac{1}{8} - \frac{x}{8}\cos 2x$

(d) $\frac{1}{8} - \frac{x}{8}\sin 2x$

93. To remove any second degree term from the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

where $a^2 + h^2 + b^2 \neq 0$, what kind of rigid motion do we need ?

(a) Translation of axes

(b) Rotation of axes

(c) Combination of translation and rotation

(d) None of the above

94. For what value of the constant b the equation

$$6x^2 + xy + by^2 + 2x - 31y - 20 = 0$$

may represent a pair of straight lines ?

(a) –12

(b) 12

(c) 0

(d) None of the above

95. If the circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch each other, then

(a) $\frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{c^2}$

(b) $\frac{1}{b^2} - \frac{1}{a^2} = \frac{1}{c^2}$

(c) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

(d) None of the above

96. The straight line $y = mx + C$ is a tangent to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

if and only if

(a) $C=\pm\sqrt{a^2m^2-b^2}$

(b) $C=\pm\sqrt{a^2m^2+b^2}$

(c) $C=\pm\sqrt{b^2-a^2m^2}$

(d) None of the above

97. The coordinates of the radical centre of the set of circles

$$x^2 + y^2 + x + 2y + 3 = 0$$

$$x^2 + y^2 + 2x + 4y + 5 = 0$$

$$x^2 + y^2 - 7x - 8y - 9 = 0$$

is

(a) (0, 0)

(b) $\left(-\frac{3}{2}, -\frac{3}{2}\right)$

(c) $\left(-\frac{2}{3}, -\frac{2}{3}\right)$

(d) None of the above

98. Which of the following statements is false for a rectangular hyperbola ?

(a) its equation can be reduced to the form $xy = 1$

(b) its asymptotes are at right angles

(c) its eccentricity is $\sqrt{2}$

(d) it has no conjugate hyperbola

99. What is the equation of the plane passing through the point (2, 1, –1) and is orthogonal to each of the planes

$$x - y + z = 1$$

$$3x + 4y - 2z = 0 \ ?$$

(a) $2x - 5y - 7z = 6$

(b) $5x - 7y - 2z = 5$

(c) $7x - 2y - 5z = 17$

(d) None of the above

100. What are the coordinates of the point where the straight line joining the points (2, –3, 1) and (3, –4, –5) cuts the plane $3x + y + z = 8$?

(a) (0, 3, 5)

(b) (2, 1, 1)

(c) (1, –2, 7)

(d) None of the above

SPACE FOR ROUGH WORK