

# Graduate Aptitude Test in Engineering 2017

Question Paper Name: Physics 5th Feb 2017  
Subject Name: Physics  
Duration: 180  
Total Marks: 100



## Organizing Institute: Indian Institute of Technology Roorkee



**Question Number : 1****Correct : 1 Wrong : -0.33**

Identical charges  $q$  are placed at five vertices of a regular hexagon of side  $a$ . The magnitude of the electric field and the electrostatic potential at the centre of the hexagon are respectively

(A) 0,0

(B)  $\frac{q}{4\pi\epsilon_0 a^2}, \frac{q}{4\pi\epsilon_0 a}$

(C)  $\frac{q}{4\pi\epsilon_0 a^2}, \frac{5q}{4\pi\epsilon_0 a}$

(D)  $\frac{\sqrt{5}q}{4\pi\epsilon_0 a^2}, \frac{\sqrt{5}q}{4\pi\epsilon_0 a}$

**Question Number : 2****Correct : 1 Wrong : 0**

A parallel plate capacitor with square plates of side 1 m separated by 1 micro meter is filled with a medium of dielectric constant of 10. If the charges on the two plates are 1 C and -1 C, the voltage across the capacitor is \_\_\_\_\_ kV. (up to two decimal places). ( $\epsilon_0 = 8.854 \times 10^{-12}$  F/m)

**Question Number : 3****Correct : 1 Wrong : 0**

Light is incident from a medium of refractive index  $n = 1.5$  onto vacuum. The smallest angle of incidence for which the light is not transmitted into vacuum is \_\_\_\_\_ degrees. (up to two decimal places).

**Question Number : 4****Correct : 1 Wrong : 0**

A monochromatic plane wave in free space with electric field amplitude of 1 V/m is normally incident on a fully reflecting mirror. The pressure exerted on the mirror is \_\_\_\_\_  $\times 10^{-12}$  Pa. (up to two decimal places) ( $\epsilon_0 = 8.854 \times 10^{-12}$  F/m).

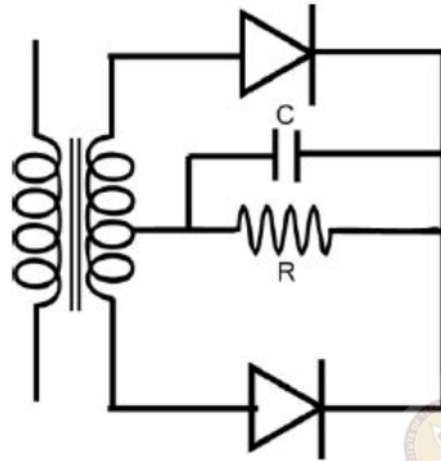
**Question Number : 5****Correct : 1 Wrong : 0**

The best resolution that a 7 bit A/D convertor with 5 V full scale can achieve is \_\_\_\_\_ mV. (up to two decimal places).

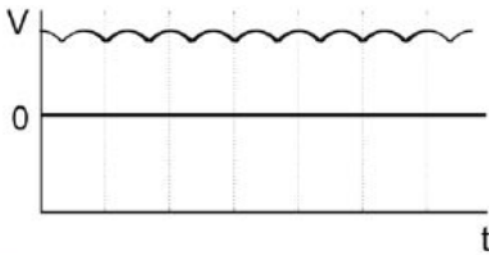
Question Number : 6

Correct : 1 Wrong : -0.33

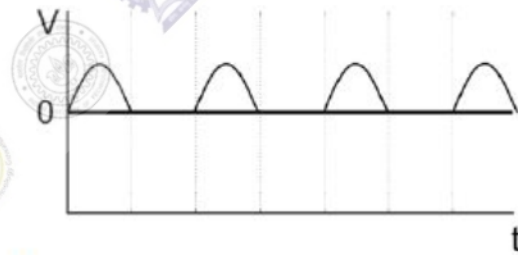
In the figure given below, the input to the primary of the transformer is a voltage varying sinusoidally with time. The resistor R is connected to the centre tap of the secondary. Which one of the following plots represents the voltage across the resistor R as a function of time?



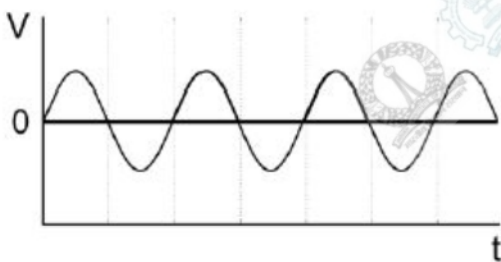
(A)



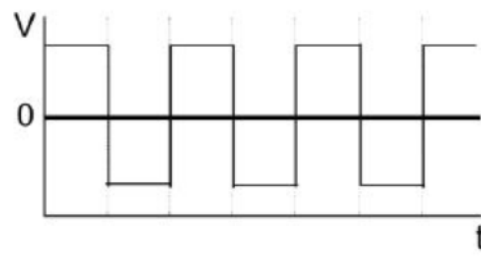
(B)



(C)



(D)



Question Number : 7

Correct : 1 Wrong : 0

The atomic mass and mass density of Sodium are 23 and  $0.968 \text{ g cm}^{-3}$ , respectively. The number density of valence electrons is  $\text{_____} \times 10^{22} \text{ cm}^{-3}$ . (Up to two decimal places.)  
(Avogadro number,  $N_A = 6.022 \times 10^{23}$ ).



**Question Number : 8****Correct : 1 Wrong : -0.33**

Consider a one-dimensional lattice with a weak periodic potential  $U(x) = U_0 \cos\left(\frac{2\pi x}{a}\right)$ . The gap at the edge of the Brillouin zone  $\left(k = \frac{\pi}{a}\right)$  is:

- (A)  $U_0$                       (B)  $\frac{U_0}{2}$                       (C)  $2U_0$                       (D)  $\frac{U_0}{4}$

**Question Number : 9****Correct : 1 Wrong : -0.33**

Consider a triatomic molecule of the shape shown in the figure below in three dimensions. The heat capacity of this molecule at high temperature (temperature much higher than the vibrational and rotational energy scales of the molecule but lower than its bond dissociation energies) is:



- (A)  $\frac{3}{2}k_B$                       (B)  $3k_B$                       (C)  $\frac{9}{2}k_B$                       (D)  $6k_B$

**Question Number : 10****Correct : 1 Wrong : -0.33**

If the Lagrangian  $L_0 = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2q^2$  is modified to  $L = L_0 + \alpha q\left(\frac{dq}{dt}\right)$ , which one of the following is TRUE?

- (A) Both the canonical momentum and equation of motion do not change  
 (B) Canonical momentum changes, equation of motion does not change  
 (C) Canonical momentum does not change, equation of motion changes  
 (D) Both the canonical momentum and equation of motion change

**Question Number : 11****Correct : 1 Wrong : 0**

Two identical masses of 10 gm each are connected by a massless spring of spring constant 1 N/m. The non-zero angular eigenfrequency of the system is \_\_\_\_\_ rad/s. (up to two decimal places).



**Question Number : 12**

**Correct : 1 Wrong : -0.33**

The phase space trajectory of an otherwise free particle bouncing between two hard walls elastically in one dimension is a

- (A) straight line      (B) parabola      (C) rectangle      (D) circle

**Question Number : 13**

**Correct : 1 Wrong : -0.33**

The Poisson bracket  $[x, xp_y + yp_x]$  is equal to

- (A)  $-x$       (B)  $y$       (C)  $2p_x$       (D)  $p_y$

**Question Number : 14**

**Correct : 1 Wrong : -0.33**

The wavefunction of which orbital is spherically symmetric:

- (A)  $p_x$       (B)  $p_y$       (C)  $s$       (D)  $d_{xy}$

**Question Number : 15**

**Correct : 1 Wrong : 0**

The contour integral  $\oint \frac{dz}{1+z^2}$  evaluated along a contour going from  $-\infty$  to  $+\infty$  along the real axis and closed in the lower half-plane by a half circle is equal to \_\_\_\_\_. (up to two decimal places).

**Question Number : 16**

**Correct : 1 Wrong : 0**

The Compton wavelength of a proton is \_\_\_\_\_ fm. (up to two decimal places).

( $m_p = 1.67 \times 10^{-27}$  kg,  $h = 6.626 \times 10^{-34}$  Js,  $e = 1.602 \times 10^{-19}$  C,  $c = 3 \times 10^8$  ms $^{-1}$ )

**Question Number : 17**

**Correct : 1 Wrong : -0.33**

Which one of the following conservation laws is violated in the decay  $\tau^+ \rightarrow \mu^+ \mu^+ \mu^-$

- (A) Angular momentum
- (B) Total Lepton number
- (C) Electric charge
- (D) Tau number

**Question Number : 18**

**Correct : 1 Wrong : -0.33**

Electromagnetic interactions are :

- (A) C conserving
- (B) C non-conserving but CP conserving
- (C) CP non-conserving but CPT conserving
- (D) CPT non-conserving

**Question Number : 19**

**Correct : 1 Wrong : -0.33**

A one dimensional simple harmonic oscillator with Hamiltonian  $H_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2$  is subjected to a small perturbation,  $H_1 = \alpha x + \beta x^3 + \gamma x^4$ . The first order correction to the ground state energy is dependent on

- (A) only  $\beta$
- (B)  $\alpha$  and  $\gamma$
- (C)  $\alpha$  and  $\beta$
- (D) only  $\gamma$

**Question Number : 20**

**Correct : 1 Wrong : -0.33**

For the Hamiltonian  $H = a_0 I + \vec{b} \cdot \vec{\sigma}$  where  $a_0 \in \mathbb{R}$ ,  $\vec{b}$  is a real vector,  $I$  is the  $2 \times 2$  identity matrix, and  $\vec{\sigma}$  are the Pauli matrices, the ground state energy is

- (A)  $|\mathbf{b}|$
- (B)  $2a_0 - |\mathbf{b}|$
- (C)  $a_0 - |\mathbf{b}|$
- (D)  $a_0$

**Question Number : 21**

**Correct : 1 Wrong : -0.33**

The coefficient of  $e^{ikx}$  in the Fourier expansion of  $u(x) = A \sin^2(\alpha x)$  for  $k = -2\alpha$  is

- (A)  $A/4$
- (B)  $-A/4$
- (C)  $A/2$
- (D)  $-A/2$

**Question Number : 22**

**Correct : 1 Wrong : -0.33**

The degeneracy of the third energy level of a 3-dimensional isotropic quantum harmonic oscillator is

- (A) 6 (B) 12 (C) 8 (D) 10

**Question Number : 23**

**Correct : 1 Wrong : -0.33**

The electronic ground state energy of the Hydrogen atom is  $-13.6 \text{ eV}$ . The highest possible electronic energy eigenstate has an energy equal to

- (A) 0 (B) 1 eV (C)  $+13.6 \text{ eV}$  (D)  $\infty$

**Question Number : 24**

**Correct : 1 Wrong : -0.33**

A reversible Carnot engine is operated between temperatures  $T_1$  and  $T_2$  ( $T_2 > T_1$ ) with a photon gas as the working substance. The efficiency of the engine is

- (A)  $1 - \frac{3T_1}{4T_2}$   
(B)  $1 - \frac{T_1}{T_2}$   
(C)  $1 - \left(\frac{T_1}{T_2}\right)^{3/4}$   
(D)  $1 - \left(\frac{T_1}{T_2}\right)^{4/3}$

**Question Number : 25**

**Correct : 1 Wrong : -0.33**

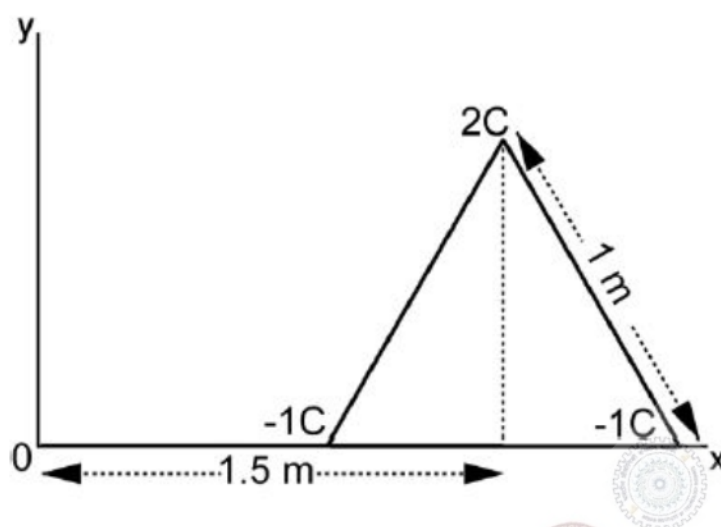
In the nuclear reaction  ${}^{13}\text{C}_6 + \nu_e \rightarrow {}^{13}\text{N}_7 + X$ , the particle  $X$  is

- (A) an electron (B) an anti-electron (C) a muon (D) a pion



**Question Number : 26****Correct : 2 Wrong : 0**

Three charges (2 C, -1 C, -1 C) are placed at the vertices of an equilateral triangle of side 1m as shown in the figure. The component of the electric dipole moment about the marked origin along the  $\hat{y}$  direction is \_\_\_\_\_ C m.

**Question Number : 27****Correct : 2 Wrong : -0.66**

An infinite solenoid carries a time varying current  $I(t) = At^2$ , with  $A \neq 0$ . The axis of the solenoid is along the  $\hat{z}$  direction.  $\hat{r}$  and  $\hat{\theta}$  are the usual radial and polar directions in cylindrical polar coordinates.  $\vec{B} = B_r \hat{r} + B_\theta \hat{\theta} + B_z \hat{z}$  is the magnetic field at a point outside the solenoid.

Which one of the following statements is true?

- (A)  $B_r = 0, B_\theta = 0, B_z = 0$
- (B)  $B_r \neq 0, B_\theta \neq 0, B_z = 0$
- (C)  $B_r \neq 0, B_\theta \neq 0, B_z \neq 0$
- (D)  $B_r = 0, B_\theta = 0, B_z \neq 0$

**Question Number : 28****Correct : 2 Wrong : 0**

A uniform volume charge density is placed inside a conductor (with resistivity  $10^{-2} \Omega \text{m}$ ). The charge density becomes  $1/(2.718)$  of its original value after time \_\_\_\_\_ femto seconds. (up to two decimal places) ( $\epsilon_0 = 8.854 \times 10^{-12} \text{F/m}$ )

**Question Number : 29**

**Correct : 2 Wrong : 0**

Water freezes at  $0^\circ\text{C}$  at atmospheric pressure ( $1.01 \times 10^5\text{ Pa}$ ). The densities of water and ice at this temperature and pressure are  $1000\text{ kg/m}^3$  and  $934\text{ kg/m}^3$  respectively. The latent heat of fusion is  $3.34 \times 10^5\text{ J/kg}$ . The pressure required for depressing the melting temperature of ice by  $10^\circ\text{C}$  is \_\_\_\_\_ GPa. (up to two decimal places)

**Question Number : 30**

**Correct : 2 Wrong : -0.66**

The minimum number of NAND gates required to construct an OR gate is:

- (A) 2                      (B) 4                      (C) 5                      (D) 3

**Question Number : 31**

**Correct : 2 Wrong : 0**

Consider a 2-dimensional electron gas with a density of  $10^{19}\text{ m}^{-2}$ . The Fermi energy of the system is \_\_\_\_\_ eV (up to two decimal places).

( $m_e = 9.31 \times 10^{-31}\text{ kg}$ ,  $h = 6.626 \times 10^{-34}\text{ Js}$ ,  $e = 1.602 \times 10^{-19}\text{ C}$ )

**Question Number : 32**

**Correct : 2 Wrong : 0**

The total energy of an inert-gas crystal is given by  $E(R) = \frac{0.5}{R^{12}} - \frac{1}{R^6}$  (in eV), where R is the inter-atomic spacing in Angstroms. The equilibrium separation between the atoms is \_\_\_\_\_ Angstroms. (up to two decimal places).



**Question Number : 33****Correct : 2 Wrong : -0.66**

Consider  $N$  non-interacting, distinguishable particles in a two-level system at temperature  $T$ . The energies of the levels are  $0$  and  $\varepsilon$ , where  $\varepsilon > 0$ . In the high temperature limit ( $k_B T \gg \varepsilon$ ), what is the population of particles in the level with energy  $\varepsilon$ ?

- (A)  $\frac{N}{2}$                       (B)  $N$                       (C)  $\frac{N}{4}$                       (D)  $\frac{3N}{4}$

**Question Number : 34****Correct : 2 Wrong : 0**

A free electron of energy  $1 \text{ eV}$  is incident upon a one-dimensional finite potential step of height  $0.75 \text{ eV}$ . The probability of its reflection from the barrier is \_\_\_\_\_ (up to two decimal places).

**Question Number : 35****Correct : 2 Wrong : -0.66**

Consider a one-dimensional potential well of width  $3 \text{ nm}$ . Using the uncertainty principle ( $\Delta x \cdot \Delta p \geq \hbar/2$ ), an estimate of the minimum depth of the well such that it has at least one bound state for an electron is ( $m_e = 9.31 \times 10^{-31} \text{ kg}$ ,  $h = 6.626 \times 10^{-34} \text{ Js}$ ,  $e = 1.602 \times 10^{-19} \text{ C}$ ):

- (A)  $1 \mu\text{eV}$                       (B)  $1 \text{ meV}$                       (C)  $1 \text{ eV}$                       (D)  $1 \text{ MeV}$

**Question Number : 36****Correct : 2 Wrong : 0**

Consider a metal with free electron density of  $6 \times 10^{22} \text{ cm}^{-3}$ . The lowest frequency electromagnetic radiation to which this metal is transparent is  $1.38 \times 10^{16} \text{ Hz}$ . If this metal had a free electron density of  $1.8 \times 10^{23} \text{ cm}^{-3}$  instead, the lowest frequency electromagnetic radiation to which it would be transparent is \_\_\_\_\_  $\times 10^{16} \text{ Hz}$ . (up to two decimal places).

**Question Number : 37****Correct : 2 Wrong : 0**

An object travels along the  $x$ -direction with velocity  $c/2$  in a frame  $O$ . An observer in a frame  $O'$  sees the same object travelling with velocity  $c/4$ . The relative velocity of  $O'$  with respect to  $O$  in units of  $c$  is \_\_\_\_\_. (up to two decimal places).



**Question Number : 38**

**Correct : 2 Wrong : 0**

The integral  $\int_0^{\infty} x^2 e^{-x^2} dx$  is equal to \_\_\_\_\_. (up to two decimal places).

**Question Number : 39**

**Correct : 2 Wrong : 0**

The imaginary part of an analytic complex function is  $v(x, y) = 2xy + 3y$ . The real part of the function is zero at the origin. The value of the real part of the function at  $1+i$  is \_\_\_\_\_. (up to two decimal places).

**Question Number : 40**

**Correct : 2 Wrong : -0.66**

Let  $X$  be a column vector of dimension  $n > 1$  with at least one non-zero entry. The number of non-zero eigenvalues of the matrix  $M = XX^T$  is

- (A) 0                      (B)  $n$                       (C) 1                      (D)  $n - 1$

**Question Number : 41**

**Correct : 2 Wrong : -0.66**

$J^P$  for the ground state of the  $^{13}\text{C}_6$  nucleus is

- (A)  $1^+$   
(B)  $\frac{3^-}{2}$   
(C)  $\frac{3^+}{2}$   
(D)  $\frac{1^-}{2}$

**Question Number : 42**

**Correct : 2 Wrong : 0**

A uniform solid cylinder is released on a horizontal surface with speed 5 m/s without any rotation (slipping without rolling). The cylinder eventually starts rolling without slipping. If the mass and radius of the cylinder are 10 gm and 1 cm respectively, the final linear velocity of the cylinder is \_\_\_\_\_ m/s. (up to two decimal places).

**Question Number : 43**

**Correct : 2 Wrong : 0**

The energy density and pressure of a photon gas are given by  $u = aT^4$  and  $P = u/3$ , where  $T$  is the temperature and  $a$  is the radiation constant. The entropy per unit volume is given by  $\alpha aT^3$ . The value of  $\alpha$  is \_\_\_\_\_. (up to two decimal places).

**Question Number : 44**

**Correct : 2 Wrong : -0.66**

Which one of the following gases of diatomic molecules is Raman, infrared, and NMR active?

(A)  $^1\text{H}-^1\text{H}$

(B)  $^{12}\text{C}-^{16}\text{O}$

(C)  $^1\text{H}-^{35}\text{Cl}$

(D)  $^{16}\text{O}-^{16}\text{O}$

**Question Number : 45**

**Correct : 2 Wrong : 0**

The  $\pi^+$  decays at rest to  $\mu^+$  and  $\nu_\mu$ . Assuming the neutrino to be massless, the momentum of the neutrino is \_\_\_\_\_ MeV/c. (up to two decimal places)  
( $m_\pi = 139 \text{ MeV}/c^2$ ,  $m_\mu = 105 \text{ MeV}/c^2$ ).

**Question Number : 46**

**Correct : 2 Wrong : -0.66**

Using Hund's rule, the total angular momentum quantum number  $J$  for the electronic ground state of the nitrogen atom is

(A) 1/2

(B) 3/2

(C) 0

(D) 1

**Question Number : 47****Correct : 2 Wrong : -0.66**

Which one of the following operators is Hermitian?

(A)  $i \frac{(p_x x^2 - x^2 p_x)}{2}$

(B)  $i \frac{(p_x x^2 + x^2 p_x)}{2}$

(C)  $e^{i p_x a}$

(D)  $e^{-i p_x a}$

**Question Number : 48****Correct : 2 Wrong : -0.66**

The real space primitive lattice vectors are  $\vec{a}_1 = a \hat{x}$  and  $\vec{a}_2 = \frac{a}{2}(\hat{x} + \sqrt{3} \hat{y})$ . The reciprocal space unit vectors  $\vec{b}_1$  and  $\vec{b}_2$  for this lattice are, respectively

(A)  $\frac{2\pi}{a}(\hat{x} - \frac{\hat{y}}{\sqrt{3}})$  and  $\frac{4\pi}{a\sqrt{3}}\hat{y}$

(B)  $\frac{2\pi}{a}(\hat{x} + \frac{\hat{y}}{\sqrt{3}})$  and  $\frac{4\pi}{a\sqrt{3}}\hat{y}$

(C)  $\frac{2\pi}{a\sqrt{3}}\hat{x}$  and  $\frac{4\pi}{a}(\frac{\hat{x}}{\sqrt{3}} + \hat{y})$

(D)  $\frac{2\pi}{a\sqrt{3}}\hat{x}$  and  $\frac{4\pi}{a}(\frac{\hat{x}}{\sqrt{3}} - \hat{y})$

**Question Number : 49****Correct : 2 Wrong : -0.66**

Consider two particles and two non-degenerate quantum levels 1 and 2. Level 1 always contains a particle. Hence, what is the probability that level 2 also contains a particle for each of the two cases: (i) when the two particles are distinguishable and (ii) when the two particles are bosons?

(A) (i) 1/2 and (ii) 1/3

(B) (i) 1/2 and (ii) 1/2

(C) (i) 2/3 and (ii) 1/2

(D) (i) 1 and (ii) 0



**Question Number : 50****Correct : 2 Wrong : 0**

A person weighs  $w_p$  at Earth's north pole and  $w_e$  at the equator. Treating the Earth as a perfect sphere of radius 6400 km, the value  $100 \times (w_p - w_e) / w_p$  is \_\_\_\_\_. (up to two decimal places). (Take  $g = 10 \text{ ms}^{-2}$ ).

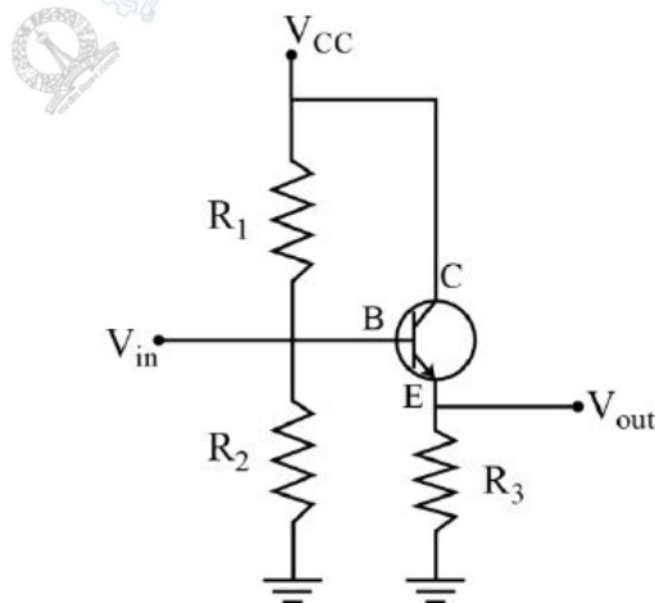
**Question Number : 51****Correct : 2 Wrong : -0.66**

The geometric cross-section of two colliding protons at large energies is very well estimated by the product of the effective sizes of each particle. This is closest to

- (A) 10 b                      (B) 10 mb                      (C) 10  $\mu$ b                      (D) 10 pb

**Question Number : 52****Correct : 2 Wrong : 0**

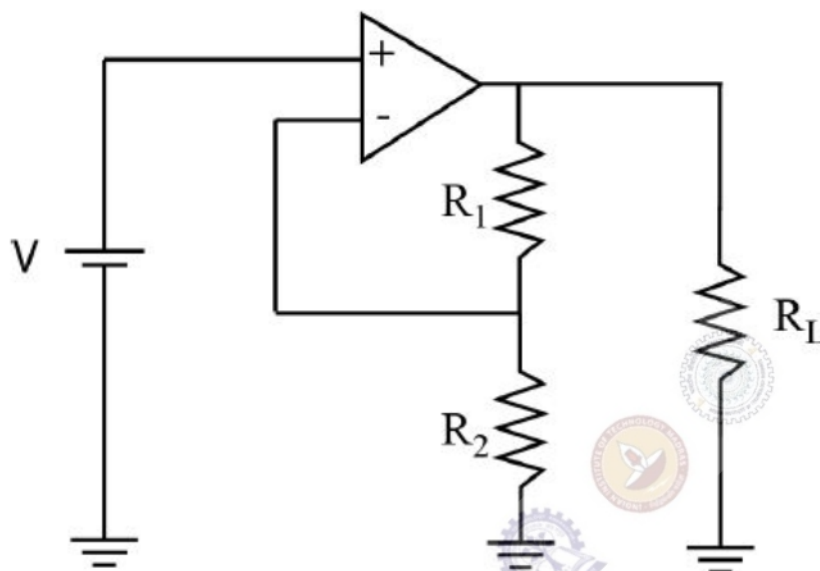
For the transistor amplifier circuit shown below with  $R_1 = 10 \text{ k}\Omega$ ,  $R_2 = 10 \text{ k}\Omega$ ,  $R_3 = 1 \text{ k}\Omega$ , and  $\beta = 99$ . Neglecting the emitter diode resistance, the input impedance of the amplifier looking into the base for small ac signal is \_\_\_\_\_  $\text{k}\Omega$ . (up to two decimal places).



**Question Number : 53**

**Correct : 2 Wrong : 0**

Consider an ideal operational amplifier as shown in the figure below with  $R_1 = 5 \text{ k}\Omega$ ,  $R_2 = 1 \text{ k}\Omega$ ,  $R_L = 100 \text{ k}\Omega$ . For an applied input voltage  $V = 10 \text{ mV}$ , the current passing through  $R_2$  is \_\_\_\_\_  $\mu\text{A}$ . (up to two decimal places).



**Question Number : 54**

**Correct : 2 Wrong : 0**

Consider the differential equation  $dy/dx + y \tan(x) = \cos(x)$ . If  $y(0) = 0$ ,  $y(\pi/3)$  is \_\_\_\_\_. (up to two decimal places).

**Question Number : 55**

**Correct : 2 Wrong : 0**

Positronium is an atom made of an electron and a positron. Given the Bohr radius for the ground state of the Hydrogen atom to be 0.53 Angstroms, the Bohr radius for the ground state of positronium is \_\_\_\_\_ Angstroms. (up to two decimal places).

**Question Number : 56**

**Correct : 1 Wrong : -0.33**

The ninth and the tenth of this month are Monday and Tuesday \_\_\_\_\_.

- (A) figuratively      (B) retrospectively      (C) respectively      (D) rightfully

**Question Number : 57**

**Correct : 1 Wrong : -0.33**

It is \_\_\_\_\_ to read this year's textbook \_\_\_\_\_ the last year's.

- (A) easier, than      (B) most easy, than      (C) easier, from      (D) easiest, from

**Question Number : 58**

**Correct : 1 Wrong : -0.33**

A rule states that in order to drink beer, one must be over 18 years old. In a bar, there are 4 people. P is 16 years old, Q is 25 years old, R is drinking milkshake and S is drinking a beer. What must be checked to ensure that the rule is being followed?

- (A) Only P's drink  
(B) Only P's drink and S's age  
(C) Only S's age  
(D) Only P's drink, Q's drink and S's age

**Question Number : 59**

**Correct : 1 Wrong : -0.33**

Fatima starts from point P, goes North for 3 km, and then East for 4 km to reach point Q. She then turns to face point P and goes 15 km in that direction. She then goes North for 6 km. How far is she from point P, and in which direction should she go to reach point P?

- (A) 8 km, East      (B) 12 km, North      (C) 6 km, East      (D) 10 km, North





**Question Number : 64**

**Correct : 2 Wrong : -0.66**

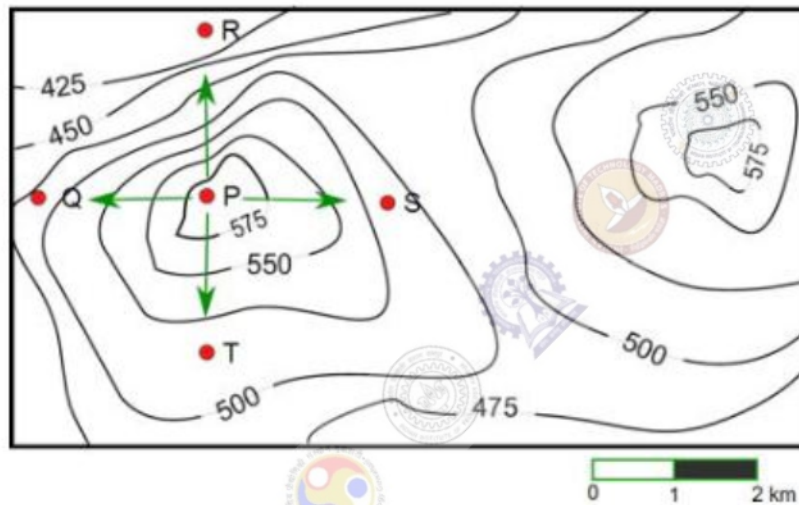
The number of 3-digit numbers such that the digit 1 is never to the immediate right of 2 is

- (A) 781                      (B) 791                      (C) 881                      (D) 891

**Question Number : 65**

**Correct : 2 Wrong : -0.66**

A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot.



Which of the following is the steepest path leaving from P?

- (A) P to Q                      (B) P to R                      (C) P to S                      (D) P to T